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Jay Smallwood

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The Paper

RADIO FREQUENCY INTERFERENCE REMOVAL THROUGH THE APPLICATION OF SPATIAL FILTERING TECHNIQUES ON THE PARKES MULTIBEAM RECEIVER

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Figure: The Astronomical Journal, 140:2086–2094, 2010 December

Outline

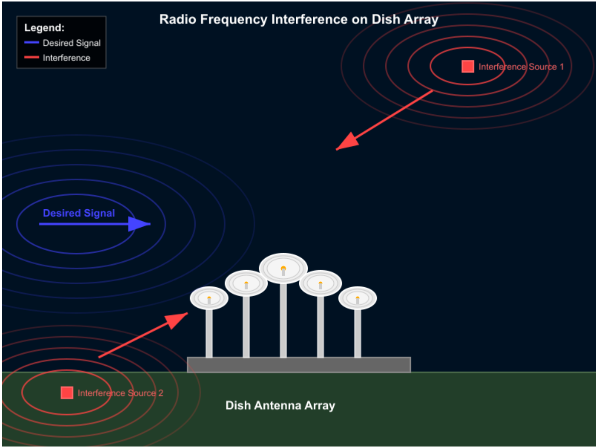
What is Spatial Filtering?

Definition

Spatial filtering is the study of using a known array spatial geometry to enhance or suppress signals arriving from specific directions to the array.

The Problem

- Radio Frequency Interference (RFI) is a problem! How can we recover as much of the celestial signal as possible?



Credit: Claude for base SVG code

The Problem

- Radio Frequency Interference (RFI) is a problem! How can we recover as much of the celestial signal as possible?

This Paper

- How can we extend existing theory of spatial filtering for single antenna arrays to multibeam receivers?
- Uses Murriyang 20 cm multibeam receiver and spatial filtering techniques to recover from RFI-contaminated data 72% of the SNR of interference-free region.

Where This Sits in the Literature

A Crash Course in Spatial Filtering

Definition

Spatial filtering is the study of using a known array spatial geometry to enhance or suppress signals arriving from specific directions to the array.

A Crash Course in Spatial Filtering

Credit: https://www.luisllamas.es/en/arduino-exponential-low-pass/

Spatial Filtering Playbook

1. Take complex voltage data from n antennas and T time samples to create a data matrix X with shape $n \times T$.

Example

$$\mathbf{X} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1T} \\ a_{21} & a_{22} & \cdots & a_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nT} \end{bmatrix} \quad \in \mathbb{C}^{n \times T}$$

Spatial Filtering Playbook

1. Take complex voltage data from n antennas and T time samples to create a data matrix X with shape $n \times T$.
2. Form the covariance matrix $\mathbf{C} = \frac{1}{T-1}\mathbf{X}\mathbf{X}^{\mathrm{H}}$ which is $n \times n$ Hermitian.

Example

$$\mathbf{C} = \frac{1}{T-1}\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \overline{\sigma_{12}} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\sigma_{1n}} & \overline{\sigma_{2n}} & \cdots & \sigma_n^2 \end{bmatrix}$$

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3. Decompose the matrix using eigendecomposition into the form $\mathbf{C} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}}$.

Example

$$\mathbf{C} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \begin{bmatrix} {\mathbf{u}_1}^H \\ {\mathbf{u}_2}^H \\ \vdots \\ {\mathbf{u}_n}^H \end{bmatrix}$$

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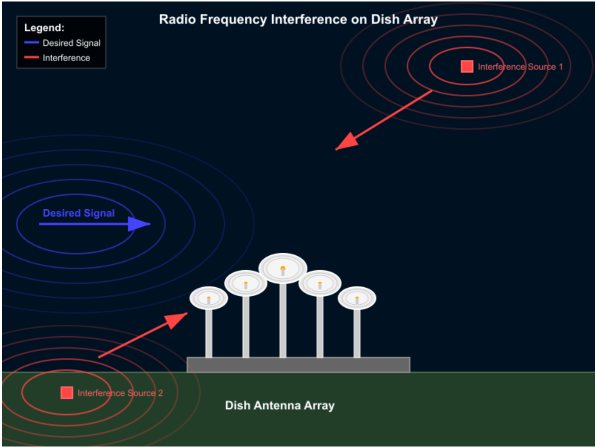
3. Decompose the matrix using eigendecomposition into the form $\mathbf{C} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}}$.
4. Use this to adjust the covariance matrix in order to remove interference:
   4.1 Nulling eigenvalues,
   4.2 Shrinking eigenvalues,
   4.3 Project data into a subspace orthogonal to the interferers.

Important Assumptions!

1. The interferer is much stronger than the celestial signal of interest and arrives from a different direction.



Credit: Claude for base SVG code

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1. The interferer is much stronger than the celestial signal of interest and arrives from a different direction.
2. The Fourier spectrum of measured signal with q interferers at antenna i is

$$S_i(f) = \underbrace{\overbrace{g_{A_i}}^{\text{Gain}} A_i(f)}_{\text{Celestial}} + \underbrace{\sum_q g_{I_q} I_q(f)}_{\text{Interference}} + \underbrace{N_i(f)}_{\text{Noise}}.$$

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3. The power of the noise $V[N_i] = \sigma^2$ is approximately equal for each antenna. If not true use noise whitening to correct.

$$\mathbf{C} = \mathbf{U} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \mathbf{U}^{\mathrm{H}}$$

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3. The power of the noise $V[N_i] = \sigma^2$ is approximately equal for each antenna. If not true use noise whitening to correct.
4. The signal components (Celestial, Interference, Noise) are all independent. Therefore we can decompose $\mathbf{C}$ into $\mathbf{C} = \underbrace{\mathbf{C_A}}_{\text{Celestial}} + \underbrace{\mathbf{C_I}}_{\text{Interference}} + \underbrace{\mathbf{C_N}}_{\text{Noise}}$.

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5. The astronomical signal is insignificant over the integration time so that $\mathbf{C} \approx \mathbf{C_I} + \mathbf{C_N}$.

Covariance Matrix Blocks

Under these assumptions we can decompose the covariance matrix further into blocks:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_I & \mathbf{U}_N \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda_I} + \sigma^2 \mathrm{I}_q & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathrm{I}_{p-q} \end{bmatrix} \begin{bmatrix} \mathbf{U}_I{}^{\mathrm{H}} \\ \mathbf{U}_N{}^{\mathrm{H}} \end{bmatrix}$$

where I represents interferers and N represents uncorrelated systematic noise.

Observed Eigenvalues

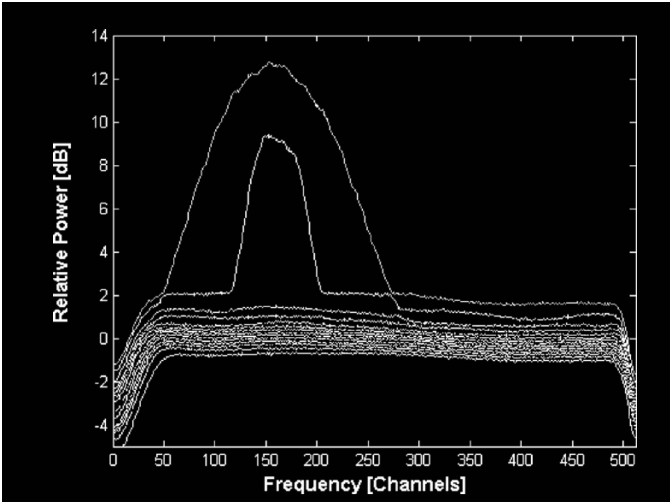

Figure 2. Eigenvalues from a power spectrum snapshot from observations of Vela pulsar at Parkes. The observation was taken at a center frequency of 1440 MHz, with an 8 MHz bandwidth. The 26 eigenvalues are from a

Methods of RFI Removal - Eigenvalue Nulling

Set the first q eigenvalues to zero and reconstruct the covariance matrix:

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Corrected:

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{U}_I & \mathbf{U}_N \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathrm{I}_{p-q} \end{bmatrix} \begin{bmatrix} \mathbf{U}_I{}^{\mathrm{H}} \\ \mathbf{U}_N{}^{\mathrm{H}} \end{bmatrix}$$

Methods of RFI Removal - Eigenvalue Shrinking

Shrink the first q eigenvalues to be the same size as the last $p-q$ eigenvalues $\tilde{\lambda} = \frac{\sum_{j=q+1}^{p} \lambda_j}{p-q}, \ i = 1, ..., q$.

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_I & \mathbf{U}_N \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda_I} + \sigma^2 \mathrm{I}_q & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathrm{I}_{p-q} \end{bmatrix} \begin{bmatrix} \mathbf{U}_I{}^{\mathrm{H}} \\ \mathbf{U}_N{}^{\mathrm{H}} \end{bmatrix}$$

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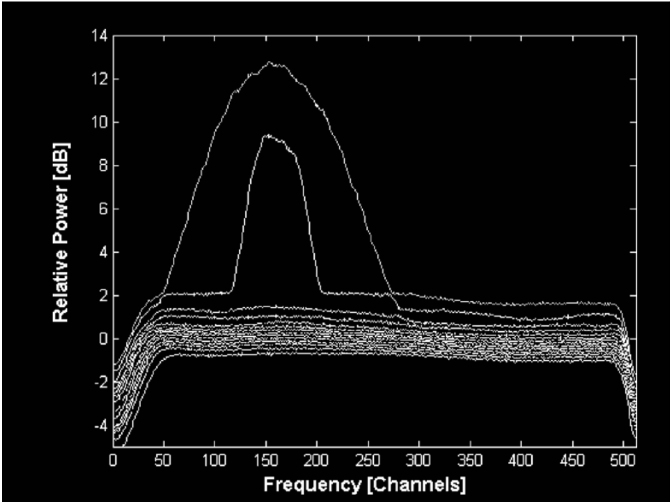
Experiment Design

- Used the Murriyang 20 cm multibeam receiver to observe the Vela pulsar.
- RFI present at 1438.5 MHz from two interferers.
- Compare the signal to noise of the pulsar averaged over 4 pulses between four methods:
  1. No correction,
  2. Eigenvalue nulling,
  3. Eigenvalue shrinking,
  4. Long-term subspace projection of short-term data (Foreshadowing!).

Observed Eigenvalues



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Be Careful Blindly Applying This!

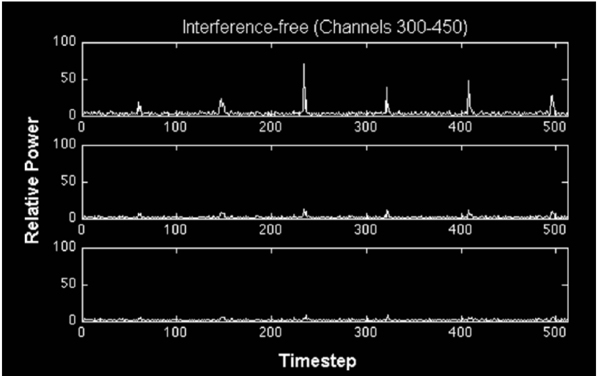


Figure 3. Comparison of the Vela pulsar detection strength in RFI non-affected spectral channels (channels 300–450), using cross-polarization data from the central beam, and after applying different RFI correction methods. Top: no correction applied. Middle: replacing the two primary eigenvalues with a noise variance estimate. Bottom: nulling the two primary eigenvalues.

Figure: Applying blindly to pulsar data can destroy the pulsar signal.

Interference-contaminated Results

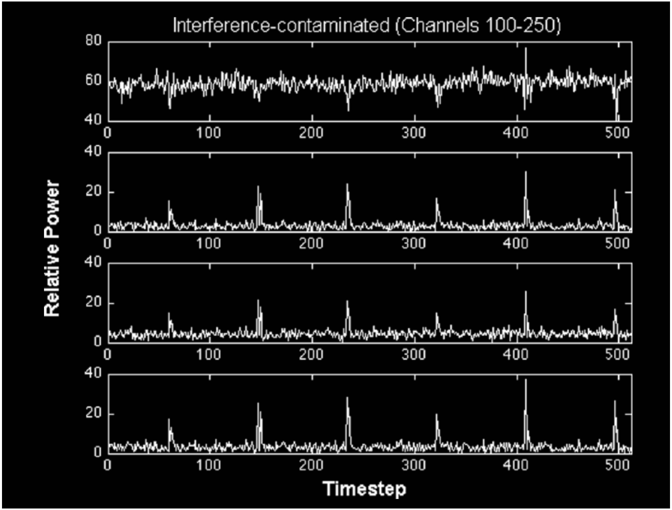


Figure: Top - No correction. Top-middle - eigenvalue shrinking. Bottom-middle - eigenvalue nulling. Bottom - primary & secondary eigenvalue shrinking only for correct region.

Interference-contaminated Results

Table 1
Vela Single Pulse S/N for Different Correction Methods, Using Cross-polarization Data

Method	Channel Range	S/N Pre-correction	S/N Post-correction
1	300–450	28.91	28.91
2	300–450	28.91	9.81
3	300–450	28.91	5.74
1	100–250	3.04	3.04
2	100–250	3.04	15.71
3	100–250	3.04	10.77
4	100–250	3.04	16.54

Notes. The S/N values are an average, taken over four pulses. Method 1: no correction. Method 2: replacing the two primary eigenvalues with a noise variance estimate. Method 3: nulling the two primary eigenvalues. Method 4: replacing the first eigenvalue in channels 46–250 and the second eigenvalue in channels 114–205 with a noise estimate computed for each frequency channel.

Long-term covariance, short-term corrections

- To avoid accidentally destroying pulses - integrate the covariance matrix over 1s and then adjust data on a 1ms timescale.
- Integrated covariance matrix is:

$$\bar{\mathbf{C}} = \bar{\mathbf{P}}_{\mathbf{N}}\bar{\mathbf{C}}\bar{\mathbf{P}}_{\mathbf{N}} + \bar{\mathbf{P}}_{\mathbf{I}}\bar{\mathbf{C}}\bar{\mathbf{P}}_{\mathbf{I}}$$

- This yields two long-term filters (NB: **not** equivalent):

$$\tilde{\mathbf{C}_{\mathbf{N}}} = \bar{\mathbf{P}}_{\mathbf{N}}\mathbf{C}\bar{\mathbf{P}}_{\mathbf{N}}$$
$$\tilde{\mathbf{C}_{\mathbf{I}}} = \mathbf{C} - \bar{\mathbf{P}}_{\mathbf{I}}\mathbf{C}\bar{\mathbf{P}}_{\mathbf{I}}$$

where **C** is the covariance matrix of 1 ms of data.

Methods of RFI Removal - Subspace Projection

Example

Take $\tilde{\mathbf{C}}_I$ as an example:

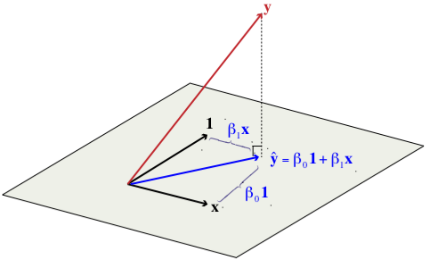
$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{\mathbf{U}}_I & \bar{\mathbf{U}}_N \end{bmatrix} \bar{\boldsymbol{\Lambda}} \begin{bmatrix} \bar{\mathbf{U}}_I{}^{\mathrm{H}} \\ \bar{\mathbf{U}}_N{}^{\mathrm{H}} \end{bmatrix}$$

$$\tilde{\mathbf{C}}_I = \mathbf{C} - \bar{\mathbf{P}}_\mathbf{I}\mathbf{C}\bar{\mathbf{P}}_\mathbf{I}$$

where $\bar{\mathbf{P}}_\mathbf{I} = \bar{\mathbf{U}}_\mathbf{I}\bar{\mathbf{U}}_\mathbf{I}{}^{\mathrm{H}}$.

- Projection operator: $P = X(X^{\mathrm{H}}X)^{-1}X^{\mathrm{H}}$
- Think regression! $H = X(X^TX)^{-1}X^T$.
- In our case, $\bar{\mathbf{P}}_I = \bar{\mathbf{U}}_I \underbrace{(\bar{\mathbf{U}}_I{}^{\mathrm{H}}\bar{\mathbf{U}}_I)^{-1}}_{=\mathrm{I}} \bar{\mathbf{U}}_I{}^{\mathrm{H}} = \bar{\mathbf{U}}_I\bar{\mathbf{U}}_I{}^{\mathrm{H}}$ due to orthonormal eigenvectors.

Methods of RFI Removal - Subspace Projection



Credit: https://sakai.unc.edu/access/content/group/2842013b-58f5-4453-aa8d-3e01bacbfc3d/public/Ecol562_Spring2012/images/lectures/lecture1/projection3.png

- Projection operator: $P = X(X^{\mathsf{H}}X)^{-1}X^{\mathsf{H}}$
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Methods of RFI Removal - Subspace Projection

Example

Take $\tilde{\mathbf{C}}_I$ as an example:

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$$\tilde{\mathbf{C}}_I = \mathbf{C} - \bar{\mathbf{P}}_\mathbf{I}\mathbf{C}\bar{\mathbf{P}}_\mathbf{I}$$

where $\bar{\mathbf{P}}_\mathbf{I} = \bar{\mathbf{U}}_\mathbf{I}\bar{\mathbf{U}}_\mathbf{I}{}^{\mathrm{H}}$.

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Long-term filter results

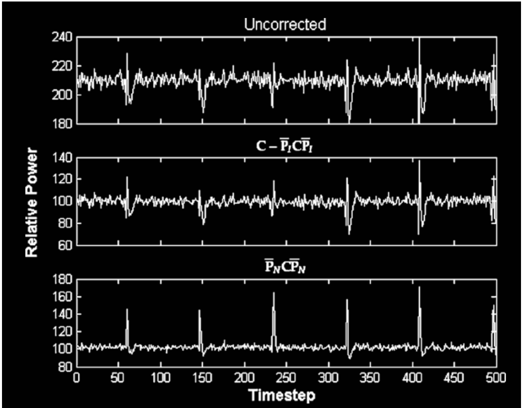

Figure 7. Interference contaminated region: detected pulsar power as a function of time for an average of spectral channels 100–250, using one polarization from the central beam. The spectra were integrated for 1 ms, and the filter calculated over 1 s. Top: the pulsar was detected from a series of spectra with no correction applied. Middle: detection after calculating the interference by projecting onto the interference subspace then subtracting away the estimate (Equation (25)). Bottom: detection after projecting directly onto the noise space (Equation (24)).

Long-term filter results

Table 2
Vela Single Pulse S/N for Different Filtering Methods,
Using Cross-polarization Data

Method	Channel Range	S/N Pre-correction	S/N Post-correction
1	300–450	28.91	28.91
2	300–450	28.91	31.03
3	300–450	28.91	32.55
1	100–250	3.04	3.04
2	100–250	3.04	19.56
3	100–250	3.04	20.86

Notes. The S/N values are an average, taken over four pulses. Method 1: no correction. Method 2: filter based on interference space projection. Method 3: filter based on noise space projection.

Alternatives to long-term filter

- Using statistical tests to derive number of interferers.
- If interference not present - no need to apply filter.
- Can also develop theory on the direction of the eigenvalues - if from boresight then likely to be celestial source.

Conclusions

- Spatial filtering is a powerful method for reducing RFI contamination of data. Recovered pulses with 72% of the SNR of inteference-free region.
- Eigenvalue shrinking is often a more powerful method than eigenvalue nulling but may destroy pulsar signals.
- Long-term subspace projection performs better than eigenvalue methods in both interference-contaminated and interference-free regions.

Appendix: Applying Corrected Covariance Matrix

Applying Corrected Covariance Matrix to Data

If $\mathbf{X}$ is the data matrix with covariance matrix $\mathbf{C}$ and corrected covariance matrix $\tilde{\mathbf{C}}$ then the corrected data $\tilde{\mathbf{X}}$ is:

$$\tilde{\mathbf{X}} = \mathbf{X}\mathbf{C}^{-0.5}\tilde{\mathbf{C}}^{0.5}$$